MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

EEM1026 – ENGINEERING MATHEMATICS II

(All Sections / All Groups)

7 MARCH 2017

9 a.m. – 11 a.m.

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of 6 **pages** (including cover page) with 4 Questions only.

2. Attempt all the questions. All questions carry equal marks and the distribution of marks for each question is given.

2. Please write all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

3. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

(a) By using the method of undetermined coefficients, solve the following inhomogeneous differential equation.

$$y'' - 5y' + 6y = -3\sin 2x$$

[11 marks]

(b) Consider the solution of $y'' - 5xy' + 4y = 0$ in the form of power series in x about $x_0 = 0$, i.e., $y = \sum_{n=0}^{\infty} c_n x^n$. Find the first six nonzero terms of this series solution. [14 marks]

Question 2

(a)(i) A random sample of 100 car owners in a city shows that a car is driven on average 23500 km/year with a standard deviation of 3900 km. Assume the distribution of measurements to be approximately normal. Based on this statement, answer the following parts a and b.

a. Construct a 99% confidence interval for the average number of kilometres an automobile is driven annually in the city. [5 marks]
b. What can we conclude with 99% confidence about the possible size of our error if we estimate the average number of kilometres driven by car owners in the city to be 23500 km/year? [2 mark]

(a)(ii) Test the hypothesis that the average content of containers of a particular lubricant is 10 litres if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3 and 9.8 liters. Use a 0.01 level of significance and assume that the distribution of contents is normal. [7 marks]

(b) Solve the following difference equation:

$$y_{k+2} - 5y_{k+1} + 6y_k = 1 \quad ; \quad y_0 = 0, y_1 = 1.$$

[11 marks]

Continued...

Question 3

(a) Given $f(t)=\begin{cases}0, & t<2\\ 2, & 2\le t<6\\ 0, & t\ge 6\end{cases}$

Determine the Laplace transform of $f(t)$. [4 marks]

(b) Find the following inverse Laplace transform by using the convolution theorem.

$$L^{-1}\left[\frac{1}{s^2(s+2)^2}\right]$$

HINT: $L^{-1}\{F(s)G(s)\}=\int_0^t f(t-\tau)g(\tau)d\tau$ [8 marks]

(c) Using Fourier sine transform, solve the following differential equation

$$u''(x)-3u(x)=10e^{-x}$$

Given that $u(0)=0$ and $u(\infty)$ is bounded. [13 marks]

Question 4

(a) By using D'Alembert formula: $u(x,t)=\frac{1}{2}[f(x-ct)+f(x+ct)]+\frac{1}{2c}\int_{x-ct}^{x+ct} g(\zeta)\,d\xi$,

solve:

PDE: $9u_{xx}=u_{tt}\quad -\infty<x<\infty;\quad t>0$

$u(x,0)=\cos x\quad ;\quad u_t(x,0)=e^{2x}\quad ;-\infty<x<\infty$

Simplify your answer. [10 marks]

(b) Solve by using separation of variables:

$xw_x+w_x-w_t+tw=0$ subject to $w(x,0)=2(x+1)^3$. [15 marks]

Continued...

APPENDIX

Table I: Laplace transform for some of function $f(t)$

$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$
1	$1/s$
t	$1/s^2$
$t^n (n = 1, 2, 3, \ldots)$	$n!/s^{n+1}$
e^{at}	$\frac{1}{s-a}$
te^{at}	$\frac{1}{(s-a)^2}$
$t^{n-1}e^{at}$	$\frac{(n-1)!}{(s-a)^n}$, $n = 1,2,\ldots\ldots$
cos at	$\frac{s}{s^2+a^2}$
sin at	$\frac{a}{s^2+a^2}$
cosh at	$\frac{s}{s^2-a^2}$
sinh at	$\frac{a}{s^2-a^2}$
$u(t - a)$	$\frac{e^{-as}}{s}$, $a \geq 0$
$f(t - a)\, u(t - a)$	$e^{-as}\mathcal{L}(f)$
$f(t)\, \delta(t - a)$	$e^{-as} f(a)$
$f'(t)$	$s\mathcal{L}(f) - f(0)$
$f''(t)$	$s^2\mathcal{L}(f) - sf(0) - f'(0)$

Continued....

Table II: Table of Fourier Transform

$f(x)$	$F(w) = F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx$
$\frac{1}{x^2 + a^2} \quad (a > 0)$	$\sqrt{\frac{\pi}{2}} \frac{e^{-a\lvert w\rvert}}{a}$
$H(x)\, e^{-ax} \quad (\text{Re } a > 0)$	$\frac{1}{\sqrt{2\pi}} \frac{1}{(a + iw)}$
$H(-x)\, e^{-ax} \quad (\text{Re } a > 0)$	$\frac{1}{\sqrt{2\pi}} \frac{1}{(a - iw)}$
$e^{-a\lvert x\rvert} \quad (a > 0)$	$\frac{1}{\sqrt{2\pi}} \frac{2a}{(w^2 + a^2)}$
e^{-x^2}	$\frac{1}{\sqrt{2}} e^{-\frac{w^2}{4}}$
$\frac{1}{2a\sqrt{\pi}} e^{-\frac{x^2}{(2a)^2}} \quad (a > 0)$	$\frac{1}{\sqrt{2\pi}} e^{-a^2 w^2}$
$\frac{1}{\sqrt{\lvert x\rvert}}$	$\frac{1}{\sqrt{\lvert w\rvert}}$
$e^{-a\frac{\lvert x\rvert}{\sqrt{2}}} \sin\left(\frac{a}{\sqrt{2}}\lvert x\rvert + \frac{\pi}{4}\right) \quad (a > 0)$	$\frac{1}{\sqrt{2\pi}} \frac{2a^3}{(a^4 + w^4)}$
$H(x + a) - H(x - a)$	$\frac{1}{\sqrt{2\pi}} \frac{2 \sin aw}{w}$
$\delta(x - a)$	$\frac{1}{\sqrt{2\pi}} e^{-iaw}$

Continued....

Table III: Table of z- Transform.

$\{x_k\}$	$F(z)$
e^{-ak}	$\frac{z}{z-e^{-a}}, \lvert z\rvert > e^{-a}$
a^k	$\frac{z}{z-a}, \lvert z\rvert > \lvert a\rvert$
ka^k	$\frac{az}{(z-a)^2}$
k^2a^k	$\frac{az(z+a)}{(z-a)^3}$
$Z\{x_{k+1}\}$	$z\,Z\{x_k\} - zx_0$
$Z\{x_{k+2}\}$	$z^2\,Z\{x_k\} - z^2x_0 - zx_1$

End of paper.